ARMY PUBLIC SCHOOL JAMMU CANTT
HOLIDAY HOMEWORK

SUBJECT: MATHS **CLASS:XII**

CHAPTER: 5. CONTINUITY AND DIFFERENTIABILITY

1. Do the miscellaneous examples and miscellaneous exercise of this chapter.

2. Discuss the continuity of the function f(x) = $\begin{cases} \frac{x}{|x|} \text{ , if } x \neq 0 \\ 1 \text{ , if } x = 0 \end{cases}$

3. Let f(x) = $\begin{cases} \frac{1-cos4x}{x^2} \;\; if\; x < 0 \\ a \quad \text{if } x = 0 \\ \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4} \;\; if\; x > 0 \end{cases}$

Determine the value of a , so that f(x) is continuous at x = 0.

4. If y = a (1 + cos θ) $and\; x = a\,(\theta - sin\theta), then\; find\; \frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{2}$

5. Discuss the continuity of the function f(x) = $\begin{cases} \frac{x}{|x|+2x^2} , x \neq 0 \text{ at } x = 0 \\ 2 \quad , x = 0 \end{cases}$

6. If cosy = x cos(a + y) and cosa $\neq$ 1 , $then\; prove\; that\; \frac{dy}{dx} = \frac{cos^2(a+y)}{sina}$

7. $If\; x\sqrt{1+y} + y\sqrt{1+x} = 0\; and\; x \neq y, then\; prove\; that\; \frac{dy}{dx} = \frac{-1}{(1+x)^2}$

8. For what values of a and b , the function f defined as f(x) = $\begin{cases} 3ax + b \text{ , if } x < 1 \text{ is continuous at } x = 1? \\ 10 \quad \text{, if } x = 1 \\ 3ax - 3b \text{ , if } x > 1 \end{cases}$

9. $If\; x^y + y^x = a^b, then\; find\; \frac{dy}{dx}$

10. Using Langrange's mean value theorem , find a point on the curve y = $\sqrt{x-2}$ defined on the interval [2 , 3] , where the tangent is parallel to the chord joining the end points of the curve.

11. If y = ($\cot^{-1} x$)^2 , $then\; show\; that$ (1 + x^2)$^2 . \frac{d^2y}{dx^2} + 2x\,(1 + x^2)\frac{dy}{dx} = 2.$

12. If $(cosx)^y = (cosy)^x, then\; find\; \frac{dy}{dx}$.

13. For what value of k , is the function f(x) = $\begin{cases} \frac{1-cos4x}{8x^2} , if\; x \neq 0 \text{ continuous at } x = 0? \\ k \quad \text{, if } x = 0 \end{cases}$

14. If f(x) and g(x) are two functions derivable in [a , b] such that f(a) = 4 , f(b) = 10 , g(a) = 1 and g(b) = 3 , then show that for a < c < b , $f'(c) = 3\,g'(c)$.

15. Differentiate $tan^{-1}\left[\frac{a\,cosx - b\,sinx}{b\,cosx + a\,sinx}\right], \frac{-\pi}{2} < x < \frac{\pi}{2}\; and\; tanx > -1\; w.r.t\; x.$

16. Verify the hypothesis and conclusion of Lagrange's mean value theorem for the function f(x) = $\frac{1}{4x-1}$, $1 \leq x \leq 4$.

17. Verify Rolle's theorem for the function f(x) = $\log\left(\frac{x^2+ab}{(a+b)x}\right)\; in\; [a,b], where\; 0 < a < b.$

18. Discuss the continuity of the function f(x) = $\begin{cases} \frac{3}{2} - x \, , if \ \frac{1}{2} \leq x < 1 \\ \frac{3}{2} \, , if \ x = 1 \\ \frac{3}{2} + x \, , if \ 1 < x \leq 2 \end{cases}$ at x = 1.

19. If f(x) = $\begin{cases} \frac{cos^2 x - sin^2 x - 1}{\sqrt{x^2+1}-1} \, , x \neq 0 \\ \text{a} \quad , \text{x} = 0 \end{cases}$ is continuous at x = 0 , then find the value of a.

20. Find $\frac{dy}{dx}$, $when \ y = \sqrt{a + \sqrt{a + \sqrt{a + x^2}}}$, $where \ a \ is \ a \ constant$

21. Differentiate $cos^{-1}\left[\frac{1-x^2}{1+x^2}\right] \ w.r.t. \ tan^{-1}\left[\frac{3x-x^3}{1-3x^2}\right]$

22. If x = sint , y = sinkt , then show that (1 - x^2) $\frac{d^2y}{dx^2} - x\frac{dy}{dx} + k^2 y = 0$.

23. Show that the function f(x) = $|$ x - 1 $|$ + $|$ x + 1 $|$, $\forall \ x \in R$, $is \ not \ differentiable \ at \ the \ points \ x = -1 \ and \ x = 1$

24. If y = $e^{m \sin^{-1} x}$, then show that (1 - x^2) $\frac{d^2y}{dx^2} - x\frac{dy}{dx} - m^2 y = 0$.

25. If f(x) = $\sqrt{x^2+1}$, $g(x) = \frac{x+1}{x^2+1} \ and \ h(x) = 2x - 3$, $then \ find \ f'[h'\{g'(x)\}]$.

26. Find the value of k for which

f(x) = $\begin{cases} \frac{\sqrt{1+kx} - \sqrt{1-kx}}{x} \quad , \text{ if } -1 \leq x < 0 \\ \frac{2x+1}{2x-1} , \qquad \text{if } 0 \leq x < 1 \end{cases}$

is continuous at x=0.

Ans. $k = -\frac{1}{2}$

27. Find the value of a for which the function f is defined as

f(x) = $\begin{cases} \text{a} \sin \frac{\pi}{2}(x+1) \ , \text{ if } x \leq 0 \\ \frac{tanx - sinx}{x^3} \quad , \text{ if } x > 0 \end{cases}$

is continuous at x=0.

Ans. $a = \frac{1}{2}$

28. Find the values of and b such that the following function f(x) is a continuous function

$$f(x) = \begin{cases} 5, & \text{if } x \leq 2 \\ ax+b, & \text{if } 2 < x < 10 \\ 21, & \text{if } x \geq 10 \end{cases}$$

Ans. a=2, b =1

29. Find the relationship between a and b, so that the function f defined by

$$f(x) = \begin{cases} ax+1, & \text{if } x \leq 3 \\ ax+b, & \text{if } x > 3 \end{cases}$$

is continuous at x=3.

Ans. 3a -3b =2

30.Show that the function f(x) is defined by

$$f(x) = \begin{cases} \frac{sinx}{x} + \cos x, & \text{if } x > 0 \\ 2, & \text{if } x = 0 \\ \frac{4(1-\sqrt{(1-x)}}{x}, & \text{if } x < 0 \end{cases}$$

is continuous at x=0.

31. If f(x) defined by the following, is continuous at x=0, then find the values of a, b and c.

$$f(x) = \begin{cases} \frac{\sin(a+1)x+sinx}{x}, & \text{if } x < 0 \\ c, & \text{if } x = 0 \\ \frac{\sqrt{x+bx^2}-\sqrt{x}}{bx^{\frac{3}{2}}}, & \text{if } x > 0 \end{cases}$$

Ans. $a = -\frac{3}{2}, c = \frac{1}{2}, b \epsilon R - \{0\}$

CHAPTER: 3 & 4 . MATRICES AND DETERMINANTS

1. Do the miscellaneous examples and miscellaneous exercise of this chapter.

2.Find inverse (if exists) of $\begin{bmatrix} 6 & 4 \\ 9 & 6 \end{bmatrix}$ by using elementary row transformation method

3. Given two matrix A = $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and B = $\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ Verify that BA = 6I , use the result to solve the system x-y =3 , 2x+ 3y+ 4z = 17 , y + 2 Z =7

4. Two schools A and B decided to award prizes to their students for three values honesty (x) and punctuality (y) and obedience (Z).School A decided to award a total of Rs 15,000 for the three values to 4,3 and 2 students respectively. While school B decided to award rs 19,000/- for the three values to 5.4 and 3 students respectively . If all three prizes together amount to rs 5,000 then

(i) Represent the above situation by a matrix equation and linear equation using matrix multiplication.
(ii) Which value you prefer to be rewarded most and why?

Ans:

School	Honesty(x)	Punctuality(y)	Obedience(Z)	Prize(Rs)
A	4	3	2	15,000
B	5	4	3	19,000
	1	1	1	5,000

5.A factory makes three products P,Q and R the table shows the units of labour , materials and other items needed to produce one of each product

P	4	3	2+a
Q	5	2+b	3
R	2	5	4

Represent this data by a matrix A. Given that labor cost Rs 10 per units, materials Rs 4 per unit and other items Rs 6 per units. represent this by a column matrix B given that , the total cost of the product P is Rs 82 and the product Q is Rs 88 respectively. find the value of a and b.

6.Using properties of determinants , prove that:

$$\begin{vmatrix} a+b & b+c & c+a \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix} = 2 \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

7.Prove that $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$ = $(a+b+c)^3$

8. Show that :

$$\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix} = 3(a+b+c)(ab+bc+ca)$$

9. Prove that $\begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix} = 0$

10. Prove that $\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix} = (bc+ca+ab)^3$

11. Prove that $\begin{vmatrix} (b+c)^2 & ab & ca \\ ab & (a+c)^2 & bc \\ ac & bc & (a+b)^2 \end{vmatrix} = 2abc\,(a+b+c)^3$

12. If A and B are square matrices of same order and B is a skew symmetric matrix , then show that $A'BA$ is a skew symmetric matrix.

13. Find the value of x,if $\begin{bmatrix} 1 & x & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = O$ (Ans. x = -2 , -14)

14. Using properties of determinants , prove that $\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix} = a^3+b^3+c^3-3abc$

15. Let f(x) = x^2-5x+6. Find f(A) , if A = $\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$ (Ans $\begin{bmatrix} 1 & -1 & -3 \\ -1 & -1 & -10 \\ -5 & 4 & 4 \end{bmatrix}$)

16. For what value of x , the matrix A is singular , if A = $\begin{bmatrix} 1+x & 7 \\ 3-x & 8 \end{bmatrix}$?

17. Using properties of determinants , prove that $\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3$

18. Show that the triangle ABC is an isosceles triangle if the determinant

$$\begin{vmatrix} 1 & 1 & 1 \\ 1+cosA & 1+cosB & 1+cosC \\ cos^2A+cosA & cos^2B+cosB & cos^2C+cosC \end{vmatrix} = 0$$

19. For a 3X3 matrix A, given that $|A| = 3$, then find $|adj(A)|$.

20. If a, b, c are all positive and are pth , qth and rth terms of a G.P , show that $\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix} = 0$

CHAPTER: 2. INVERSE TRIGONOMETRIC FUNCTIONS

1. Do the miscellaneous examples and miscellaneous exercise of this chapter.

2. Prove the following

$\cos(\sin^{-1}3/5 + \cot^{-1} 3/2) = \frac{6}{5\sqrt{3}}$

3.Prove that $\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right) = \frac{\pi}{4} + \frac{1}{2}\cos^{-1}x^2$

4.To prove, $\tan\left(\frac{1}{2}\sin^{-1}\frac{13}{4}\right) = \frac{4-\sqrt{7}}{3}$

5.Prove $\sin[\cot^{-1}\{\cos(\tan^{-1}x)\}] = \sqrt{\frac{x^2+1}{x^2+2}}$

6.Prove the following $\cos^{-1}x = 2\sin^{-1}\sqrt{\frac{1-x}{2}} = 2\cos^{-1}\sqrt{\frac{1-x}{2}}$

7.Solve for x : $\cos^{-1}\left(\frac{x^2-1}{x^2+1}\right) + \tan^{-1}\left(\frac{2x}{x^2-1}\right) = \frac{2\pi}{3}$

8.Solve the following equation : $\cos(\tan^{-1} x) = \sin(\cot^{-1} ¾)$

9.Solve the following equation:

$\sin^{-1}[x\sqrt{1-(1-x)^2} + (1-x)\sqrt{1-x^2}] = \cos^{-1}x$

10.Solve $\sin^{-1}x + \sin^{-1}2x = \frac{\pi}{3}$

11. Solve $\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$

12. Simplify the following

$\cos^{-1}\left(\frac{3}{5}\cos x + \frac{4}{5}\sin x\right)$

13. Simplify

$\sin^{-1}\left(\frac{\sin x+\cos x}{\sqrt{2}}\right)$, $-\frac{\pi}{4} < x < \frac{\pi}{4}$

14.Simplify

$\tan^{-1}\left(\frac{3a^2x-x^3}{a^3-3ax^2}\right)$, $\frac{-1}{\sqrt{3}} < \frac{x}{a} < \frac{1}{\sqrt{3}}$

15. Simplify

$\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$

16. Write the following function in simplest form

$\tan^{-1}\{ \sqrt{\frac{a-x}{a+x}} \}$, a <x<a

CHAPTER: 1. RELATIONS AND FUNCTIONS

1. Do the miscellaneous examples and miscellaneous exercise of this chapter.

2. Show that the relation R in R defined as R={(a,b): a≤b} is reflective and transitive but not symmetric

3. If F:x → y is a function. Define a relation R on x given by R ={(a,b): f(a) = f(b) } show that R is an equivalence relation on x

4. Prove that the the relation R on the set N×N defined by (a,b) R (c,d) $\Longleftrightarrow$ a+b =b+c for all (a,b) , (c,d) ϵ N×N is an equivalence relation.

5.Let f:N→ N be defined by f(x) = $\begin{cases} \frac{n+1}{2} ,, if\ x\ is\ odd \\ \frac{n}{2} ,, if\ x\ is\ even \end{cases}$ for all n ϵ N find whether the function f is bijective.

6.Show that f:N → N , given by

F(x) = $\{ \begin{matrix} x+1, if\ x\ is\ odd \\ x-1, if\ \ x\ is\ even \end{matrix}$

Is both one- one and onto.